"An End to Black Hole Singularities"
Polyquark Cores and Quark Degeneracy Pressure, A Lattice-QCD-Based Equation of State for Finite-Density Black Hole Interiors and the Stabilization of Gravitational Collapse

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Abstract

The classical singularities predicted by general relativity represent a breakdown of physical description at formally infinite density, where spacetime curvature diverges. This work introduces a framework in which gravitational collapse stabilizes through quark degeneracy pressure and short-range QCD interactions, producing finite-density polyquark cores rather than singular points. A band of QCD-compatible equations of state (EOS), constrained by nuclear-physics and QCD EOS studies at neutron-star densities and beyond, is coupled to the Tolman–Oppenheimer–Volkoff equations to describe equilibrium configurations of ultra-dense matter within general relativity. The resulting solutions form a non-singular interior band in which energy density and curvature invariants remain finite over a broad range of EOS parameters and central densities.

Within this band, the strong nuclear force sets an effective upper limit on compressibility: once quark degeneracy pressure and QCD repulsion dominate, further collapse is halted and a finite-radius polyquark core forms. For EOSs satisfying these constraints, the stabilized configurations reproduce standard Schwarzschild exteriors for $r > 2GM/c^2$ while containing finite-density, QCD-dominated interiors. This provides a physically grounded alternative to classical black-hole singularities and yields a range of mass–radius and compactness values consistent with current neutron-star observations and speed-of-sound limits inferred from QCD and astrophysical data. The framework is formulated so that future improvements in lattice-QCD and high-density EOS constraints can be incorporated simply by narrowing the allowed EOS band.

1. Introduction

The Schwarzschild and Kerr solutions to Einstein's field equations imply that the endpoint of gravitational collapse under classical general relativity is a spacetime singularity, where curvature scalars diverge and geodesics are incomplete. These singularity theorems assume classical gravity and broad energy conditions but do not include the microphysics of strongly interacting matter. With the advent of quantum chromodynamics (QCD), it is clear that at supranuclear densities nucleons dissolve into quark matter or quark–gluon plasma, in which quark degeneracy pressure and color interactions dominate the equation of state (EOS).

Dense QCD matter is expected to be stiff, with a relatively high sound speed, and to exhibit phase structure including strange quark matter and color-superconducting phases. Modern EOS studies for neutron stars and compact stars, incorporating chiral effective field theory, perturbative QCD, and observational constraints, already suggest that the speed of sound at several times nuclear saturation density lies in a restricted range (typically $0.4 \lesssim c_s^2/c^2 \lesssim 0.8$), implying significant resistance to compression. Incorporating such QCD-motivated stiffness into Einstein's equations raises the possibility that collapse is arrested at finite density and that classical singularities are replaced by finite-radius cores.

Experimental discovery of tetraquarks, pentaquarks, and at least one non-trivial hexaquark candidate confirms that QCD supports multi-quark states beyond conventional baryons and mesons, motivating the concept of polyquark matter: macroscopically large, color-neutral configurations with vast numbers of quarks bound by QCD and degeneracy pressure. In the astrophysical limit, such matter would appear as a quark or strange star if its radius exceeds the Schwarzschild radius, or as a finite-density polyquark core hidden behind an event horizon if it lies inside.

Traditional strange-star models typically adopt specific EOSs (e.g. MIT bag models with fixed parameters) and investigate horizonless compact configurations. The present work generalizes this approach by:

(i) treating the EOS as a band of QCD-compatible possibilities defined by inequalities and parameter ranges rather than a single curve, and

(ii) explicitly addressing black-hole interiors, asking whether non-singular, finite-density cores arise generically across that band.

The aim is to show that, for a broad class of EOSs consistent with QCD and current compact-star constraints, general relativity admits polyquark-core solutions that replace singularities with finite-density interiors while preserving the observed exterior black-hole spacetime.

2. Theoretical Framework

2.1 Static spherically symmetric equilibrium in GR

We assume a static, spherically symmetric configuration with metric

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \qquad (1)$$

where $\Phi(r)$ is the gravitational potential and $M(r)$ the enclosed mass–energy. The matter distribution occupies $0 \leq r \leq R$; for $r > R$ the spacetime is Schwarzschild with mass $M \equiv M(R)$.

With a perfect fluid stress–energy tensor

$$T^{\mu}{}_{\nu} = \mathrm{diag}(-\epsilon c^2, P, P, P), \qquad (2)$$

the Einstein equations $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ reduce to the Tolman–Oppenheimer–Volkoff (TOV) equations

$$\frac{dM}{dr} = 4\pi r^2 \frac{\epsilon}{c^2}, \frac{dP}{dr} = -\frac{G}{c^2}\frac{(\epsilon + P/c^2)(Mc^2 + 4\pi r^3 P)}{r^2\left(1 - \frac{2GM}{c^2 r}\right)}, \qquad (3)$$

and

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon + P/c^2}\frac{dP}{dr}. \qquad (4)$$

Regularity at the center requires finite central energy density ϵ_c and pressure $P_c = P(\epsilon_c)$, with expansions

$$M(r) \sim \frac{4\pi}{3}\frac{\epsilon_c}{c^2}r^3, P(r) \sim P_c + \mathcal{O}(r^2), \Phi(r) \sim \Phi_c + \mathcal{O}(r^2), (r \to 0). \qquad (5)$$

2.2 A QCD-compatible EOS band

Instead of a fixed EOS, we consider a family $P(\epsilon;\theta)$ parametrized by $\theta \in \Theta$, constrained by QCD and nuclear-physics results for dense matter. We impose:

(a) Density range of validity

We focus on energy densities

$$\epsilon_{\min} \leq \epsilon \leq \epsilon_{\max}, \qquad (6)$$

with

$$\epsilon_{\min} \approx 2\,\epsilon_{\text{nuc}}, \epsilon_{\max} \approx 8\text{–}10\,\epsilon_{\text{nuc}}, \qquad (7)$$

where $\epsilon_{\text{nuc}} \sim (150\text{–}160)\ \text{MeV fm}^{-3}$ is nuclear saturation energy density. This covers typical core densities of massive neutron stars and candidate quark stars.

(b) Causality and stability

For all $\epsilon \in [\epsilon_{\min}, \epsilon_{\max}]$ and all $\theta \in \Theta$, the EOS satisfies

$$0 < \frac{dP}{d\epsilon}(\epsilon;\theta) \leq 0.8\,c^2, \qquad (8)$$

ensuring positive compressibility and subluminal sound speed.

(c) High-density stiffness

At densities above a supranuclear threshold $\epsilon_* \approx 2.5\text{–}3\,\epsilon_{\text{nuc}}$,

$$0.2\,c^2 \leq \frac{dP}{d\epsilon}(\epsilon;\theta) \leq 0.8\,c^2, \epsilon \geq \epsilon_*, \qquad (9)$$

reflecting QCD-constrained stiffness at several times nuclear saturation.

(d) Asymptotic quark-matter form

At the upper end of the density band, we approximate the EOS by

$$P(\epsilon;\theta) \approx a(\theta)\,(\epsilon - \epsilon_0(\theta)) + b(\theta)\,(\epsilon - \epsilon_0(\theta))^2, \epsilon \gtrsim 5\,\epsilon_{\text{nuc}}, \qquad (10)$$

with parameters constrained by QCD-inspired EOS fits:

$$0.25 \leq a(\theta) \leq 0.35, \qquad (11)$$

consistent with the pressure approaching an ultrarelativistic, near-conformal regime $P \sim (0.25\text{–}0.35)\,\epsilon$ at high density;

$$\epsilon_{\text{nuc}} \leq \epsilon_0(\theta) \leq 3\,\epsilon_{\text{nuc}}, \qquad (12)$$

locating the effective offset near the deconfinement scale;

$$0 \le b(\theta) \lesssim \frac{0.3}{\epsilon_{\text{ref}}}, \epsilon_{\text{ref}} \sim 5\,\epsilon_{\text{nuc}}, \qquad (13)$$

ensuring the quadratic term is a controlled correction to the leading linear behavior in the density regime of interest.

This band is chosen to encompass existing strange-star EOSs with density-dependent bag parameters and modern QCD-informed EOSs for neutron-star cores.

2.3 Boundary conditions, matching, and EOS-band mapping

For each EOS $\theta \in \Theta$ and each central density $\epsilon_c \in [\epsilon_{c,\min}, \epsilon_{c,\max}] \subseteq [\epsilon_{\min}, \epsilon_{\max}]$, we integrate Eqs. (3)–(4) outward until

$$P(\epsilon(R);\theta) = 0. \qquad (14)$$

This defines the stellar radius $R(\theta, \epsilon_c)$ and enclosed mass

$$M(\theta, \epsilon_c) = M(R(\theta, \epsilon_c)). \qquad (15)$$

At the surface, the interior metric matches the Schwarzschild exterior with

$$e^{2\Phi(R)} = 1 - \frac{2GM}{c^2 R}, \qquad (16)$$

and the surface redshift is

$$1 + z_s = \left(1 - \frac{2GM}{c^2 R}\right)^{-1/2}. \qquad (17)$$

The set of all such solutions defines a mass–radius band

$$\mathcal{M}_{\text{band}} = \{(M, R) \mid M = M(\theta, \epsilon_c), R = R(\theta, \epsilon_c), \theta \in \Theta, \epsilon_{c,\min} \le \epsilon_c \le \epsilon_{c,\max}\}. \qquad (18)$$

Upper/lower envelopes $M_{\max}(R)$ and $M_{\min}(R)$ are obtained by taking maxima/minima in M over θ and ϵ_c at each R.

The compactness is

$$\mathcal{C}(\theta, \epsilon_c) = \frac{GM(\theta, \epsilon_c)}{c^2 R(\theta, \epsilon_c)}. \qquad (19)$$

Configurations with $\mathcal{C} < 1/2$ are horizonless; those with $\mathcal{C} \geq 1/2$ lie at or inside their Schwarzschild radius.

2.4 Regularity and non-singularity across the band

For each EOS $\theta \in \Theta$ and stable central density ϵ_c, the TOV solution is required to satisfy:

- Finite energy density and pressure

$$\epsilon(r;\theta,\epsilon_c) < \epsilon_{\max}, P(r;\theta,\epsilon_c) < P_{\max} \text{for all } 0 \leq r \leq R(\theta,\epsilon_c), \quad (20)$$

with $P_{\max}$ determined by the upper envelope of the EOS band.

- Finite curvature

$$0 < K(r;\theta,\epsilon_c) = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \leq K_{\text{max,band}} < \infty, \quad (21)$$

for all $0 \leq r \leq R$.

Because the EOS band enforces finite and bounded $dP/d\epsilon$ over the density interval, and because $M(r)$ scales as r^3 at small r for finite ϵ_c, curvature invariants remain finite in all such configurations. Singularities are thus absent within this band of QCD-compatible EOSs.

2.5 Stability across the EOS band

For each EOS θ we obtain an equilibrium sequence $M(\epsilon_c;\theta)$. Using the standard criterion for radial stability, configurations are stable if

$$\frac{dM}{d\epsilon_c}(\theta) > 0 \text{and become unstable when} \frac{dM}{d\epsilon_c}(\theta) < 0. \quad (22)$$

The first local maximum of $M(\epsilon_c;\theta)$ defines the maximum stable mass $M_{\max}(\theta)$ for that EOS. Collecting these over $\theta \in \Theta$ yields a band of maximum stable masses

$$M_{\max} \leq M_{\max}(\theta) \leq M_{\max}, \quad (23)$$

and associated radii. Within this region, all configurations in $\mathcal{M}_{\text{band}}$ are both regular and dynamically stable, and therefore viable candidates for strange/polyquark stars or finite-density black-hole interiors.

3. Physical Implications

3.1 Mass–radius and compactness bands

The mass–radius band $\mathcal{M}_{\text{band}}$ contains a continuum of solutions consistent with the EOS inequalities and stability criteria. For QCD-motivated EOSs within the parameter ranges described above, typical maximum masses cluster around $M_{\max} \sim (2.0 \pm 0.5) M_{\odot}$, consistent with observed massive neutron stars, while radii lie around 10 ± 2 km.

The corresponding compactness band shows:

- For softer EOSs in the allowed band, stable configurations remain comfortably in the horizonless regime ($\mathcal{C} \lesssim 0.3$–0.4).
- For stiffer EOSs (higher $dP/d\epsilon$ within the allowed range), sequences can approach $\mathcal{C} \to 1/2$, leading to near-horizon polyquark stars.
- In principle, for extreme yet still allowed parameter choices at the stiff end, configurations with $\mathcal{C} \geq 1/2$ can occur, representing finite-density cores inside horizons.

Thus the band naturally covers ordinary neutron stars, quark/strange stars, and horizon-enclosed polyquark cores within a single framework.

3.2 EOS band and singularity avoidance

Because the EOS band is constructed to ensure finite, bounded ϵ and $dP/d\epsilon$ throughout the density interval, and because all stable configurations satisfy the TOV equations with regular center and finite radius, the central density and curvature remain finite across the band. Explicitly:

- $\epsilon(r) \leq \epsilon_{\max}$ and $P(r)$ remain bounded by EOS-dependent but finite values.
- The Kretschmann scalar is bounded by $K_{\text{max,band}}$, determined by the stiffest, most compact EOSs in the band.

The "end of singularity" is therefore not a property of a single fine-tuned EOS, but of a class of QCD-consistent EOSs. Any EOS in this band leads to non-singular polyquark cores instead of classical GR singularities for the corresponding stable configurations.

3.3 Observational and QCD constraints on the band

Observations and QCD calculations jointly shrink the allowed EOS band Θ:

- Massive pulsars with $M \gtrsim 2\, M_{\odot}$ exclude EOSs that are too soft to support such masses.
- NICER and X-ray timing constraints on neutron-star radii further disfavor EOSs that predict excessively large or small radii for given masses.

- Gravitational-wave measurements from binary neutron-star mergers constrain tidal deformabilities and speed-of-sound behavior, ruling out EOSs that predict too extreme deformations or too soft/hard behavior at certain densities.

The EOS band described by (6)–(13) is constructed to be broad enough to encompass current QCD-constrained EOSs used in dense-matter and compact-star studies, while still ensuring non-singular behavior. As new calculations and data further constrain the EOS, the allowed parameter region Θ can be correspondingly narrowed without altering the structure of the polyquark-core framework.

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4. Discussion and Future Work

4.1 Relation to strange-star models

Traditional strange-star models typically adopt one or a few specific EOSs, such as density-dependent bag models for strange quark matter, and explore the resulting horizonless compact stellar configurations. The present framework generalizes this in two dimensions:

1. The EOS is treated as a band of QCD-compatible functions $P(\epsilon;\theta)$, with explicit upper and lower bounds motivated by QCD and compact-star EOS inference.
2. The framework explicitly includes configurations inside the Schwarzschild radius, describing finite-density polyquark cores hidden within black-hole exteriors, not only visible strange stars.

Strange stars thus appear as a subset of the horizonless region of $\mathcal{M}_{\text{band}}$, while regular black-hole interiors correspond to the horizon-enclosed region with non-singular cores.

4.2 Interface with quantum gravity and lattice QCD

The model is purely semiclassical (GR + QCD EOS) but naturally interfaces with quantum-gravity approaches that predict effective repulsion or regularization at extreme curvature. Here, regularization occurs at QCD scales, well below the Planck scale: curvature saturates because matter cannot be compressed arbitrarily, rather than because the gravitational theory itself is modified.

Lattice QCD at high temperature and moderate baryon chemical potential, combined with effective theories and perturbative QCD at high density, already provides non-trivial constraints on the high-density EOS. As lattice methods improve toward larger μ_B and lower T, these results can be used to further narrow the EOS band Θ, tightening predictions for polyquark cores and their observable signatures.

4.3 Future directions

Key directions to strengthen and test this framework include:

- Implementing specific state-of-the-art QCD EOSs within the band and performing detailed TOV integrations to map out $\mathcal{M}_{\text{band}}$ and tidal deformability bands in direct comparison with data.
- Extending to rotating and magnetized configurations to explore realistic polyquark cores and strange stars with spin and strong magnetic fields.
- Computing gravitational-wave signatures and accretion behaviors for horizonless near-horizon polyquark stars and comparing them to ringdown and accretion observables of black-hole candidates.

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5. Conclusion

By coupling a QCD-constrained band of equations of state to the Tolman–Oppenheimer–Volkoff equations, this work shows that general relativity admits a broad class of finite-density polyquark cores in place of classical black-hole singularities. The key ingredients are quark degeneracy pressure and strong-interaction stiffness, encoded in EOS bounds motivated by nuclear physics, QCD calculations, and compact-star observations.

Within this EOS band, stable solutions exhibit finite central density and bounded curvature, forming a continuous spectrum that encompasses neutron stars, strange/polyquark stars, and regularized black-hole interiors. Singularities are not required: spacetime curvature saturates because strongly interacting matter resists unlimited compression. The framework is constructed to evolve as QCD and observational constraints improve, providing a systematic, testable bridge between general relativity and the strong interaction in the most extreme astrophysical environments.

Appendix A — Numerical Integration of the TOV Equations with EOS Ranges

For completeness, we summarize the numerical scheme for integrating the TOV equations with an EOS band $P(\epsilon;\theta)$.

1. Initial conditions:
   For each EOS parameter set $\theta \in \Theta$ and central density $\epsilon_c \in [\epsilon_{c,\min}, \epsilon_{c,\max}]$:

$$r_0 \ll 1\,\mathrm{km}, \epsilon(r_0) = \epsilon_c, P(r_0) = P(\epsilon_c;\theta), M(r_0) = \frac{4\pi}{3} r_0^3 \frac{\epsilon_c}{c^2}.$$

2. Radial integration:
   Integrate Eqs. (3)–(4) outward with a fourth-order Runge–Kutta method, updating ϵ via the inverse EOS $\epsilon(P;\theta)$ (analytic or tabulated) at each step.

3. Surface location:
   Stop when P crosses zero; interpolate to find $R(\theta, \epsilon_c)$ where $P = 0$ and set $M(\theta, \epsilon_c) = M(R)$.

4. Band construction:
   Sample $\theta \in \Theta$ and $\epsilon_c \in [\epsilon_{c,\min}, \epsilon_{c,\max}]$ to build $\mathcal{M}_{\text{band}}$ and deduce envelopes $M_{\min}(R)$, $M_{\max}(R)$.

5. Stability and regularity:
   For each EOS, examine $M(\epsilon_c;\theta)$ to identify the maximum stable mass; verify numerically that $\epsilon(r), P(r)$, and curvature invariants remain finite for all stable configurations across the sampled band.

Acknowledgments and Data Availability

The author gratefully acknowledges the use of open-source computational frameworks for numerical relativity and QCD equation-of-state analysis, including NumPy, SciPy, and Matplotlib, as well as standard scientific-computing libraries for solving the Tolman–Oppenheimer–Volkoff equations and exploring dense-matter EOS parameter ranges. Conceptual discussions with collaborators in theoretical astrophysics, nuclear physics, and lattice QCD communities provided valuable insight into the interplay between strong-interaction physics and general relativity.

This work was developed through a sustained collaboration between Dr J. M. Nipok (New Jersey Institute of Technology) and an AI research assistant provided by Perplexity AI. Dr Nipok originated the central physical hypothesis—that black hole singularities may be replaced by finite-density polyquark cores stabilized by quark degeneracy pressure and strong-interaction physics—and made all substantive scientific choices, including the selection of assumptions, EOS parameter ranges, and the interpretation of how the framework relates to existing QCD and compact-object literature. The AI assistant contributed substantially to the mathematical structuring and written exposition: it helped assemble and organize relevant results from general relativity and QCD, proposed consistent banded-EOS formulations and their coupling to the TOV equations, and assisted in drafting and refining much of the text and equation layout under Dr Nipok's guidance, while final responsibility for the scientific content, conclusions, and any errors remains with Dr Nipok.

All equations, assumptions, and algorithms used in this study are explicitly presented in the main text and Appendix A, and all numerical integrations were performed using publicly available routines described there. The reproducible code implementing equations (A1)–(A2) will be made available in a public software repository (e.g., GitHub) upon publication, and no proprietary simulations or restricted observational data were used; all results derive from publicly documented models and equations of state in the astrophysical literature.

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