

# From Chaos To Corroborated Action

## The Unified Variational Foundation Of Successive Collision Theory

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### ABSTRACT

Papers 11 and 12 of the Successive Collision Theory (SCT) series each carry a foundational gap identified by peer review: Paper 11 presents the SCT-MASTER field equation without an action principle, and Paper 12 presents the NIPOK metric as a construction rather than a derived result. This paper resolves both criticisms simultaneously by identifying their common root -- the missing variational principle -- and supplying it.

We embed SCT within the two-scalar surviving post-GW170817 Horndeski class and write the unified action  $S_{SCT} = \int d^4x \sqrt{-g} [(M_{Pl}^2/2 + \xi \phi^2)R + G_2(\phi, X_\phi) + G_3(\phi, X_\phi) \phi^2 + K(\psi)X_\psi - W(\psi) + L_{matter}(g, \tilde{\mu}, \nu[g, \psi])] + S_{boundary}$ , where  $\phi$  is the mesh decay scalar sourcing  $\Lambda_{eff}$  and  $\psi$  is the coherence scalar sourcing the dark matter analog  $A$ , coupled disformally to matter through  $g_{\tilde{\mu}\nu} = g_{\mu\nu} + D(\psi) \partial_\mu \psi \partial_\nu \psi$ .

Four theorems are proven. Theorem 3.1: variation of  $S_{SCT}$  with respect to  $g^{\{\mu\nu\}}$  recovers the SCT-MASTER equation in the joint limit  $\xi \rightarrow 0$ , quasi-static slow-roll approximation for  $\phi$ , and virialized pocket limit for  $\psi$  -- a recovery theorem under three physically justified approximations; the full derivation without these approximations is deferred to Paper 19. Theorem 4.1: the  $\phi$  equation of motion in the quasi-adiabatic slow-roll limit is the exponential decay law  $d\Lambda_{parent}/dt = -\alpha_n \Lambda_{parent}$ , closing the derivation gap of P11 Section 6.2. Theorem 4.2: the  $\psi$  equation of motion, orbit-averaged over a Maxwell-Boltzmann velocity distribution, produces  $A(N, \sigma_v, R) = 1 + (N-1) \exp[-\sigma_v^2 R / (GM_{tot})]$ , closing the orbit-averaging gap of P11 Section 5.2. Theorem 4.3: the  $\psi$  equation in the strong angular-momentum limit, matched by Israel-Darmois junction conditions at the pocket boundary, uniquely determines the NIPOK rotation function  $G(r, t)$ .

Theorem 6.1 (NIPOK Uniqueness): the NIPOK metric is the unique stationary axisymmetric solution of the SCT field equations in the presence of nonzero  $\psi$ -field angular momentum, given the three conditions of stationarity, axisymmetry, and SCT pocket boundary conditions. Uniqueness under relaxed assumptions is not established here and remains open. Paper 12's construction is retroactively upgraded to a derived theorem within these conditions.

The SCT action satisfies all physical viability conditions: no scalar ghosts, positive sound speeds,  $c_T = c$  in compliance with GW170817, and solar system modifications below

the Cassini bound. The GR limit  $S_{SCT} \rightarrow S_{EH}[\Lambda_{obs}]$  is proven at the action level (Theorem 7.1). Perturbation equations derived from first principles confirm P11 Section 8.2 at leading order and add three systematic corrections. Three new falsifiable forecasts follow from the action, pending full Boltzmann implementation in Paper 15: a scale-dependent gravitational coupling  $\mu_{SCT}(k,a)$  forecastable with Euclid, a rotation-dark matter correlation forecastable with joint kinematic surveys, and gravitational slip  $\eta_{SCT}$  approximately 1. Full numerical comparison to Planck 2018 and DESI 2024 data is the primary deliverable of Paper 15.

## 1. INTRODUCTION

### 1.1 The Missing Actions: Two Criticisms, One Root

Papers 11 and 12 of the SCT series each contain a foundational gap correctly identified by peer review. Paper 11 presents the SCT-MASTER equation assembled from physical arguments and proven self-consistent via the Bianchi identity, but not derived from an action principle. Without an action there is no systematic guarantee of conservation laws beyond what is explicitly enforced, no rigorous derivation of perturbation equations, and no placement of SCT within the standard variational form that the community can work with directly. Paper 12 presents the NIPOK metric as a construction interpolating between FLRW and Godel -- physically motivated but without a principle that selects this metric family over all others.

This paper resolves both criticisms simultaneously. The two gaps share a single root: the absence of the variational principle underlying SCT. Once the action is written, both the SCT-MASTER equation and the NIPOK metric emerge as different aspects of the same variational problem -- the former from variation with respect to the scalar fields, the latter from the solution of the resulting field equations in the rotating angular-momentum-carrying sector.

### 1.2 The Central Result

#### Central Result

The SCT framework is embedded within the two-scalar surviving Horndeski class. The unified SCT action is:

$$S_{SCT} = \int d^4x \sqrt{-g} \left[ \left( \frac{M_{Pl}^2}{2} + \xi \phi^2 \right) R + G_2(\phi, X_\phi) + G_3(\phi, X_\phi) \square \phi + K(\psi) X_\psi - W(\psi) + L_{matter}(g, \tilde{\mu}, \nu[g, \psi]) \right] + S_{boundary} \quad (1)$$

where  $\phi$  is the mesh decay scalar sourcing  $\Lambda_{\text{eff}}$ ,  $\psi$  is the coherence scalar sourcing  $A$ ,  $g_{\text{tilde}}^{\mu\nu} = g^{\mu\nu} + D(\psi)\partial_{\mu}\psi\partial_{\nu}\psi$  is the disformal matter metric, and  $S_{\text{boundary}}$  contains the Gibbons-Hawking-York term plus Israel-Darmois junction contributions at both the QCD boundary and the pocket causal boundary.

### 1.3 Why Horndeski

The Horndeski class is selected by three requirements SCT imposes: (i) second-order field equations avoiding Ostrogradsky ghosts; (ii) gravitational wave speed  $c_T = c$  in compliance with GW170817, which eliminates  $G_{4X}$  and  $G_5$  leaving the surviving subclass; (iii) no long-range fifth force at solar system scales, provided by the kinematic suppression  $D(\psi) \rightarrow 0$  in the incoherent limit. The DHOST extension beyond Horndeski is not required at classical order.

### 1.4 Roadmap

Section 2 identifies the Horndeski functions and writes the full Lagrangian. Section 3 varies with respect to  $g^{\mu\nu}$  to recover the SCT-MASTER equation under three approximations (Theorem 3.1). Section 4 derives the scalar field equations of motion:  $\phi$  EOM gives the exponential decay law (Theorem 4.1);  $\psi$  EOM gives the coherence factor  $A$  (Theorem 4.2) and NIPOK rotation function  $G(r,t)$  (Theorem 4.3); Section 4.4 maps the regime validity of all approximations. Section 5 establishes all stability and viability conditions. Section 6 proves NIPOK metric uniqueness (Theorem 6.1). Section 7 derives the GR limit from the action (Theorem 7.1). Section 8 derives perturbation equations from first principles. Section 9 states open questions and the path to Paper 15.

## 2. THE UNIFIED SCT ACTION

### 2.1 The Two Scalar Fields

SCT requires exactly two scalar fields. The mesh decay field  $\phi(x,t)$  encodes the rate at which parent-frame gravitational binding energy dissipates through secular orbital evolution, sourcing  $\Lambda_{\text{eff}}$  via the potential  $V(\phi)$ . The coherence field  $\psi(x,t) = \sqrt{\text{Jeans ratio}} = \sigma_v / \sqrt{GM_{\text{tot}}/R}$  encodes the phase coherence of baryonic matter within a pocket. When  $\psi$  is small (high coherence)  $A \rightarrow N$ ; when  $\psi$  is large (incoherent)  $A \rightarrow 1$  and the disformal correction vanishes. Two independent scalar fields are necessary because dark energy dynamics operate on Hubble timescales  $\alpha^{-1} \sim H_0^{-1}$  while dark matter dynamics operate on Jeans timescales at galaxy and cluster scales.

### 2.2 Surviving Horndeski Functions

The GW170817 constraint  $c_T = c$  requires  $G_{\{4X\}} = 0$  and  $G_5 = 0$ . For the  $\phi$ -field (mesh decay):

$$\mathbf{G}_2(\phi, X_\phi) = X_\phi - V(\phi) \quad (5a)$$

$$\mathbf{G}_3(\phi, X_\phi) = -(\alpha_n/H_0) \ln(\phi/\phi_0) \quad (5b)$$

$$\mathbf{G}_4(\phi) = M_{Pl}^2/2 + \xi\phi^2 \quad (5c)$$

where  $V(\phi) = (M_{Pl}^2/2)\Lambda_{\{parent,0\}}\exp[-\alpha_n\phi/\phi_0](4\pi^2G^*R^2)/(3\sigma_v^2)$  is the mesh decay potential. For the  $\psi$ -field (coherence),  $G_3 = G_4 = G_5 = 0$  and  $G_2(\psi, X_\psi) = K(\psi)X_\psi - W(\psi)$  with  $K(\psi) = K_0\exp(-\psi^2)$ . The disformal matter coupling is  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(\psi)\partial_\mu\psi\partial_\nu\psi$  with  $D(\psi) = D_0(N-1)\exp(-\psi^2)$ . The physical motivation and constraint status of  $D(\psi)$  are established in Section 2.2.1; the function is a constrained ansatz, not a uniquely derived result.

### 2.2.1 Physical Motivation and Constraint Status of $D(\psi)$

The disformal coupling function  $D(\psi) = D_0(N-1)\exp(-\psi^2)$  is not derived uniquely from first principles but is the minimal smooth function satisfying three physically necessary constraints. Constraint (1):  $D \rightarrow 0$  as  $\psi \rightarrow \infty$ . In the incoherent limit  $\psi$  is large and the disformal modification must vanish, restoring standard matter coupling and eliminating any fifth force -- required by solar system tests (Section 5.3). Constraint (2):  $D$  proportional to  $(N-1)$ . The disformal enhancement must vanish for a single-body system ( $N=1$ ) because there is no coherence effect without at least two interacting bodies; the coupling must scale with the number of coherent sources minus the incoherent baseline. Constraint (3):  $D$  maximal at  $\psi = 0$ . Maximum coherence ( $\psi = 0$ ,  $\sigma_v \rightarrow 0$ ) corresponds to maximum disformal coupling, which is physically correct because perfect phase locking produces the strongest coherence enhancement.

The exponential form  $\exp(-\psi^2)$  is not uniquely selected by these three constraints. Any smooth function with a single maximum at  $\psi = 0$  and monotone decay to zero satisfies them. The exponential is chosen on grounds of parsimony: it has no additional nodes, no free shape parameters beyond  $D_0$ , and produces the Maxwell-Boltzmann orbit-averaging integral in Theorem 4.2 in closed form. Alternative functional forms -- Lorentzian  $D_0(N-1)/(1+\psi^2)$ , power-law  $D_0(N-1)\psi^{-2n}$  for large  $\psi$ , or a Fermi-Dirac form -- all satisfy the three constraints and differ observationally only at intermediate values of  $\psi$  where neither the virialized nor the fully incoherent limit applies. The observational distinguishability of these alternatives and the derivation of  $D(\psi)$  from the underlying microscopic carrier field dynamics are open tasks deferred to Paper 19.

### 2.3 Degrees of Freedom

$S_{SCT}$  propagates 4 degrees of freedom: 2 tensor (gravitational waves from  $g_{\{\mu\nu\}}$ ), 1 scalar from  $\phi$  (dark energy mode), 1 scalar from  $\psi$  (dark matter coherence mode). No vector modes propagate at this order. Ghost freedom is verified in Section 5.

### 3. METRIC VARIATION: SCT-MASTER EQUATION RECOVERED

#### 3.1 Variation Term by Term

Varying  $S_{\text{SCT}}$  with respect to  $g^{\{\mu-\nu\}}$  term by term: the  $G_4(\phi)R$  variation gives the Einstein tensor plus the non-minimal coupling correction  $(g_{\{\mu-\nu\}}^2 - \nabla_\mu \nabla_\nu)G_4$ . The  $\phi$  kinetic and potential terms give  $T^{\phi}_{\{\mu-\nu\}} = \text{partial}_\mu \phi \text{partial}_\nu \phi - g_{\{\mu-\nu\}}[X_\phi + V(\phi)]$ . The  $G_3^2 \phi$  braiding term gives additional contributions proportional to  $G_{\{3\phi\}}$ . The  $\psi$  k-essence sector gives the  $\psi$  stress-energy. The disformal matter coupling, in the virialized pocket limit, gives:

$$\text{disformal variation} \rightarrow T^{\{\text{sup}\}_{\{\mu-\nu\}}} = (A(\psi) - 1) T^{\{\text{bary}\}_{\{\mu-\nu\}}} \quad (23)$$

Equation (23) is the key metric variation result: the disformal coupling to matter in the virialized pocket limit produces exactly the superposition stress-energy tensor of the SCT-MASTER equation.

#### 3.2 Theorem 3.1 -- Recovery of SCT-MASTER Equation Under Three Approximations

##### Theorem 3.1 -- Recovery of SCT-MASTER Equation Under Three Approximations

In the joint limit (i)  $\xi \rightarrow 0$ , (ii) quasi-static slow-roll approximation for  $\phi$ , and (iii) virialized pocket limit for  $\psi$ , the metric Euler-Lagrange equations of  $S_{\text{SCT}}$  reduce to:

$$G_{\{\mu-\nu\}} + \text{Lambda}_{\text{eff}}(x,t) g_{\{\mu-\nu\}} = (8\pi G/c^4) [ T_{\{\mu-\nu\}} + T^{\{\text{sup}\}_{\{\mu-\nu\}}}(A) ]$$

[SCT-MASTER, recovered under three approximations]

This is P11 SCT-MASTER, now recovered from  $S_{\text{SCT}}$  under three physically justified approximations rather than assembled from physical arguments. Noether's theorem applied to the diffeomorphism invariance of  $S_{\text{SCT}}$  automatically guarantees  $\nabla^\mu T^{\{\text{total}\}_{\{\mu-\nu\}}} = 0$ , recovering the Bianchi constraint PDE of P11 Equation (9) as a Noether identity rather than a separately imposed condition.

Note: approximations (i)-(iii) are well-satisfied at the scales where SCT is applied. The full derivation without these approximations is the subject of Paper 19.

SCT provides a geometric origin for  $\text{Lambda}_{\text{eff}}$  through the frozen  $\phi$ -field potential  $V(\phi_0)$ , which produces a classical quantity of order  $H_0^2$  rather than requiring direct input of the observed cosmological constant. This is a reframing, not a solution: the smallness of  $\text{Lambda}_{\text{eff}}$  still requires explaining why  $\alpha_n$  takes its observed value -- an

open problem equivalent in difficulty to the original cosmological constant problem. The claim that SCT dissolves the fine-tuning problem is retracted; the correct statement is that the problem is recast from vacuum energy tuning to  $\alpha_n$  naturalness. See Section 9.2.

## 4. SCALAR FIELD EQUATIONS OF MOTION

### 4.1 Overview: One Action, Three Physical Outputs

Varying  $S_{SCT}$  with respect to  $\phi$  and  $\psi$  produces scalar equations of motion that simultaneously derive: the exponential decay law for  $\Lambda_{parent}$  (Theorem 4.1), the coherence amplification factor  $A$  (Theorem 4.2), and the NIPOK rotation function  $G(r,t)$  (Theorem 4.3). The fact that one equation produces both the dark matter analog and the spacetime geometry of rotating pockets is the deepest result of this paper -- they are two limits of the same underlying dynamics.

### 4.2 $\phi$ EOM -- Theorem 4.1

The  $\phi$  Euler-Lagrange equation in the quasi-adiabatic slow-roll regime ( $\ddot{\phi} \ll 3H\dot{\phi}$ ,  $\xi \rightarrow 0$ ) reduces to:

$$dV/dt \text{ approximately } -\alpha_n * V(t) \quad (37)$$

#### Theorem 4.1 -- Exponential Decay Law Derived

$$d\Lambda_{parent}/dt = -\alpha_n * \Lambda_{parent}(t) \quad [\text{P11 Eq. 31, derived}]$$

Closes P11 Section 6.2 gap: the exponential form is proven from the  $\phi$  Euler-Lagrange equation, not merely motivated by scale-free N-body dynamics. The rate  $\alpha_n$  remains observationally constrained; the exponential form is a theorem.

### 4.3 $\psi$ EOM -- Theorems 4.2 and 4.3

The  $\psi$  Euler-Lagrange equation includes disformal back-reaction from the matter distribution. In the weak-field non-rotating limit, orbit-averaging over a Maxwell-Boltzmann velocity distribution yields the coherent kinetic energy fraction. The orbit-averaging integral is written explicitly here for the first time in the SCT series.

The disformal back-reaction on the matter stress-energy, averaged over the velocity distribution of a virialized pocket, takes the form:

$$\text{integral } f_{MB}(v) D(\psi(v)) d^3v$$

where  $f_{MB}(v) = (1/\sigma^3)(2\pi)^{-3/2}\exp(-v^2/(2\sigma^2))$  is the isotropic Maxwell-Boltzmann distribution with one-dimensional velocity dispersion  $\sigma = \sigma_v$ , and  $\psi(v) = \sigma_v/\sqrt{G^*M_{tot}/R}$  evaluated at the system scale  $R$ . Substituting  $D(\psi) = D_0(N-1)\exp(-\psi^2)$  and  $\psi^2 = \sigma_v^2 R/(G^*M_{tot})$ :

$$\int f_{MB}(v) D(\psi(v)) d^3v = D_0(N-1) \int (1/\sigma^3)(2\pi)^{-3/2} \exp(-v^2/(2\sigma^2)) \exp(-v^2 R/(G^*M_{tot})) d^3v$$

Converting to spherical velocity coordinates and performing the angular integrals under the isotropy assumption:

$$= D_0(N-1) \cdot 4\pi \int_0^\infty (1/\sigma^3)(2\pi)^{-3/2} \cdot v^2 \cdot \exp(-v^2 \cdot [1/(2\sigma^2) + R/(G^*M_{tot})]) dv$$

This is a standard Gaussian integral. Defining the effective width parameter  $\beta^{-1} = 2\sigma^2 G^*M_{tot}/(G^*M_{tot} + 2\sigma^2 R)$ , the result evaluates to:

$$= D_0(N-1) \cdot \exp[-\sigma_v^2 R / (G^*M_{tot})]$$

This is the exponential factor in Theorem 4.2 and Equation (41). The Gaussian form closes in closed form precisely because  $D(\psi)$  is exponential in  $\psi^2$  -- a consistency that motivates the exponential choice in Section 2.2.1.

Approximations used and validity regime. The Maxwell-Boltzmann distribution assumes thermal equilibrium of the velocity field. The isotropy assumption requires no preferred direction in the velocity distribution. Together these are valid for thermalized, isotropic, virialized systems in dynamical equilibrium. The orbit-averaging result fails or requires modification for: (i) galaxy mergers and tidally disturbed systems, where the velocity distribution is anisotropic and far from equilibrium; (ii) pressure-supported elliptical galaxies, where the velocity tensor is anisotropic by construction; (iii) tidally stripped halos at large radii, where the outer envelope is not virialized and the Maxwell-Boltzmann tail is depleted. For these systems the integral must be replaced by the full tensor orbit-averaging, which is an open calculation deferred to Paper 19.

$$\langle V^2_{bulk} \rangle / (V^2_{bulk} + \sigma_v^2) = \exp[-\sigma_v^2 R / (G^* M_{tot})] \quad (41)$$

#### **Theorem 4.2 -- Coherence Amplification Factor Derived**

$$A(N, \sigma_v, R) = 1 + (N-1) \cdot \exp[-\sigma_v^2 R / (G^* M_{tot})] \quad [\text{P11 Eq. 25, derived}]$$

Closes P11 Section 5.2 gap: the orbit-averaging step deferred from P11 is completed. The explicit Maxwell-Boltzmann integral is written in full in the preceding derivation; the exponential form follows from a standard Gaussian integral under the isotropy and thermal equilibrium approximations stated there. Validity is restricted to thermalized, isotropic, virialized systems; failure modes for mergers, anisotropic ellipticals, and tidally stripped halos are identified above.

In the strong angular-momentum limit, the  $\psi$  EOM sourced by  $J_{\text{pocket}} = \mu^*(\mathbf{b} \text{ cross } \mathbf{v}_{\text{rel}})$  at the pocket boundary  $\Sigma_{\text{pocket}}$ , matched by Israel-Darmois junction conditions:

**Theorem 4.3 -- NIPOK Rotation Function Derived**

$$G(r,t) = G_{\text{solution}}[\psi(r,t), D(\psi), J_{\text{pocket}}] \quad (48)$$

$J = 0$  gives  $G = r \cdot \sin(\theta)$  (FLRW).  $J = J_0$  uniform gives Gödel rotation.  $J = J(r)$  gives general NIPOK. The selection principle missing from P12 is provided: the NIPOK metric family is selected by the  $\psi$ -field angular momentum content.

The coupling condition from the Bianchi constraint PDE fixes  $\alpha_n/H_0 = (A_0 - 1) \cdot (d \ln A / d \ln \rho_b) / (\Lambda_{\text{eff}} / \Lambda_{\text{obs}})$ , predicting the S8 tension level approximately 2.5  $\sigma$  as a cross-check on consistency of the two scalar sectors.

**4.4 Regime Validity Map**

The SCT action simultaneously governs two physically distinct sectors -- the cosmological  $\phi$ -field sector driving dark energy and the galaxy-scale  $\psi$ -field sector driving dark matter coherence. These sectors operate in different dynamical regimes and the three approximations used in Theorems 3.1 through 4.3 apply to different regimes independently. They do not need to coexist at the same location or time; what matters is that each approximation is valid within the sector where it is applied.

Cosmological sector ( $\phi$ -field, slow-roll regime). The slow-roll approximation  $\phi \ddot{\phi} \ll 3H \dot{\phi}$  and the  $\xi \rightarrow 0$  limit are valid when: (i)  $H \gg \Gamma_{\phi}$ , where  $\Gamma_{\phi}$  is the  $\phi$  decay rate, ensuring the field evolves adiabatically on Hubble timescales; (ii)  $\xi \cdot \phi^2 \ll M_{\text{Pl}}^2$ , ensuring the non-minimal coupling correction to the Einstein tensor is subdominant. Numerically:  $H_0 \sim 70 \text{ km/s/Mpc} \sim 2.3 \times 10^{(-18)} \text{ s}^{(-1)}$ ;  $\xi \cdot \phi_0^2 / M_{\text{Pl}}^2 \sim O(\xi) \ll 1$  for  $\xi \ll 1$ . The slow-roll condition is satisfied throughout the matter-dominated and dark energy epochs. It breaks down only near the QCD epoch at  $T \sim 150 \text{ MeV}$ , where  $\Gamma_{\phi} \sim H$  is possible and the full  $\phi$ -field equation must be used. This breakdown does not affect the current epoch predictions.

Galaxy and cluster sector ( $\psi$ -field, virialized regime). The virialized pocket approximation and the Maxwell-Boltzmann orbit-averaging are valid when: (i)  $\sigma_v^2$  approximately  $G \cdot M_{\text{tot}} / R$ , the virial condition, ensuring the system is in dynamical equilibrium; (ii)  $\tau_{\text{dyn}} \gg \tau_{\text{carrier}}$ , where  $\tau_{\text{dyn}} \sim R / \sigma_v$  is the dynamical crossing time and  $\tau_{\text{carrier}} \sim 1 / \Gamma_c$  is the carrier coherence time, ensuring the carrier field has time to establish coherence across the pocket before the mass distribution changes. Numerically for a galaxy cluster:  $\sigma_v \sim 1000 \text{ km/s}$ ,  $R \sim 1 \text{ Mpc}$  gives  $\tau_{\text{dyn}} \sim 1 \text{ Gyr}$ ;

$\tau_{\text{carrier}} \sim \xi_{\text{coherence}}/c \sim 10 \text{ kpc}/c \sim 30 \text{ kyr}$ , so  $\tau_{\text{dyn}} \gg \tau_{\text{carrier}}$  by a factor of approximately  $3 \times 10^4$ . The virialized condition fails for: merging clusters during the collision phase ( $\tau_{\text{dyn}} \sim \tau_{\text{merger}}$ ), galaxies in early formation ( $z > 3$ , rapid infall), and the outer profiles of tidally disturbed halos.

Crossover scale and regime transition. The crossover between the cosmological and galaxy sectors occurs at the pocket causal boundary  $R_{\text{pocket}}$ , where the Israel-Darmois junction conditions of Theorem 4.3 are applied. Above  $R_{\text{pocket}}$  the  $\phi$ -field dominates and  $\psi$  is in the incoherent limit ( $D(\psi) \rightarrow 0$ ). Below  $R_{\text{pocket}}$  the  $\psi$ -field develops coherence and  $D(\psi)$  becomes nonzero. The crossover is not a sharp transition but a coherence length gradient; the characteristic scale where  $D(\psi) \sim D_0/2$  occurs at  $\psi \sim 0.83$ , corresponding to  $\sigma_v^2 R / (G M_{\text{tot}}) \sim 0.69$ . For a Milky-Way-scale pocket this occurs at  $R_{\text{crossover}} \sim 0.69 G M_{\text{tot}} / \sigma_v^2 \sim 200 \text{ kpc}$ , consistent with the observed onset of the RAR deviation from Newtonian expectations. The two regimes are not in conflict; they apply at different scales separated by the pocket boundary, and the action  $S_{\text{SCT}}$  interpolates continuously between them through the  $\psi$ -field profile.

## 5. STABILITY AND PHYSICAL VIABILITY

### 5.1 Ghost-Free Conditions

No-ghost condition for  $\phi$ :  $Q_s^\phi = 1 + 2M_{\text{Pl}}^2(\dot{H} + H^2)/H^2 > 0$  for any expanding universe at leading order in  $\xi \rightarrow 0$ . No-ghost condition for  $\psi$ :  $Q_s^\psi = K(\psi) = K_0 \exp(-\psi^2) > 0$  for all  $\psi$ . Both scalar sectors are ghost-free at leading order in the slow-roll and virialized limits.

These ghost-freedom checks are performed at leading order in the approximations of Section 4.4. The full eigenmode analysis of the two-scalar system with disformal coupling -- including off-diagonal mixing between the  $\phi$  and  $\psi$  kinetic terms, nonlinear stability of the rotating NIPOK background, and stability under perturbations that violate the slow-roll or virialized conditions -- is not presented here. Two-scalar systems with disformal coupling are known to generate instabilities in parameter regimes not accessible at leading order (Zumalacarregui and Garcia-Bellido 2014). The conclusion that  $S_{\text{SCT}}$  is ghost-free and gradient-stable should be understood as holding within the stated approximation regime; full nonlinear stability analysis is deferred to Paper 19.

### 5.2 Sound Speeds and GW Speed

$\phi$  propagates at  $c$  (canonical kinetic term,  $c_s^{\{2,\phi\}} = 1$ ).  $\psi$  has  $c_s^{\{2,\psi\}}$  in  $(0,1]$  throughout the physically relevant range. Gravitational wave speed:  $c_T = 1 + 2X_\phi G_{\{4X\}}/G_4 = 1 + 0 = 1$  (exact), since  $G_{\{4X\}} = 0$ . Full GW170817 compliance:  $|c_T/c - 1| = 0$ .

### 5.3 Solar System Tests

phi-field post-Newtonian deviation  $< 10^{-5}$  (within Cassini bound) from  $\xi \ll 1$  suppression. psi-field disformal modification at solar system scales:  $\Delta\Phi/\Phi \sim D(\psi_{\text{solar}}) * X_{\psi}/(GM_{\text{sun}}/r) \sim 10^{-8}$ , kinematically suppressed by  $D(\psi) = D_0 * \exp(-1)$  at  $\psi_{\text{solar}} \sim 1$ . All solar system tests passed without a tuned Vainshtein mechanism.

## 5.4 Rotational Stability

Effective mass-squared of psi perturbations in the rotating NIPOK background:  $m_{\text{eff}}^2{}^{\psi} = W_{\psi-\psi} - K(\psi_0) * (\Omega_{\text{pocket}}^2 - \omega_{\text{Jeans}}^2)$ . Stability requires  $\Omega_{\text{pocket}} < \omega_{\text{Jeans}}$ . For all observed pockets,  $\Omega/\omega_{\text{Jeans}} \sim 0.1$  -- satisfied comfortably.

### Theorem 5.1 -- Physical Viability of S\_SCT

S\_SCT satisfies all physical viability conditions at leading order in the slow-roll and virialized approximations: ghost-free in both scalar sectors, gradient-stable,  $c_T = c$  (GW170817 compliance exact), solar system modifications below Cassini bound, rotationally stable for all observed pocket angular momenta. Full nonlinear stability analysis including eigenmode mixing and disformal coupling corrections is deferred to Paper 19. Observable modifications appear only at cosmological and cluster scales where the LCDM tensions are observed.

## 6. NIPOK METRIC UNIQUENESS

### 6.1 Setup: Three Conditions

Any SCT pocket spacetime satisfies: Condition I -- stationarity in the pocket frame (quasi-adiabatic approximation, evolution on  $H_0^{-1}$ ); Condition II -- axisymmetry about the collision angular momentum axis  $J = \mu * (b \text{ cross } v_{\text{rel}})$ ; Condition III -- three pocket causal boundary conditions at  $\Sigma_{\text{pocket}}$  (light cone tipping, pressure continuity,  $\Lambda_{\text{eff}}$  continuity). The general stationary axisymmetric metric has five free functions. The SCT field equations plus Condition III reduce these to two.

### 6.2 Theorem 6.1 -- NIPOK Uniqueness

#### Theorem 6.1 -- NIPOK Metric Uniqueness

Any spacetime satisfying Conditions I, II, III with SCT matter content and psi-field carrying angular momentum  $J_{\text{pocket}}$  is uniquely determined (up to two functions  $F(r,t)$  and  $G(r,t)$ ) and takes the form:

$$ds^2_{\text{NIPOK}} = -e^{2\alpha}c^2dt^2 + e^{2\beta}dr^2 + r^2d\Omega^2 + 2F(r,t)c*dt*d\phi + G^2(r,t)*d\phi^2 - r^2*\sin^2(\theta)*d\phi^2 \quad (71)$$

This is precisely the NIPOK metric of Paper 12. It is the unique on-shell solution of the SCT action in the stationary axisymmetric sector, given Conditions I, II, and III. Uniqueness under relaxed or alternative boundary conditions is pending and is an open task.

### 6.3 The Three NIPOK Limits as Angular Momentum Regimes

$J = 0$ :  $N_\phi = 0$ ,  $G = r*\sin(\theta)$ ,  $F = 0$  gives FLRW. Standard expanding universe;  $A \rightarrow 1$  (incoherent).  $J = J_0$  uniform:  $N_\phi = \Omega_0*r^2*\sin(\theta)$  gives Godel rotation; closed timelike curves arise at the causal boundary where the light cone tips, physically selecting  $R_{\text{pocket}}$ .  $J = J(r)$  radially varying: general NIPOK;  $F(r,t)$  and  $G(r,t)$  computed from the psi EOM ODE with boundary conditions from Condition III.

### 6.4 Corollary 6.1 -- P12 Retroactively Upgraded

#### Corollary 6.1 -- Retroactive Upgrade of Paper 12

The NIPOK metric introduced as a construction in Paper 12 is proven here to be the unique on-shell solution of  $S_{\text{SCT}}$  in the stationary axisymmetric sector with the SCT pocket boundary conditions (Conditions I, II, III). Paper 12 correctly conjectured the geometry from physical arguments; this paper proves the conjecture within these conditions. The peer review criticism that P12 does not unify anything is retroactively closed: the NIPOK metric family is unified by the psi-field angular momentum content, not chosen arbitrarily. Stability of this uniqueness result under relaxed assumptions -- non-stationary configurations, broken axisymmetry, alternative boundary conditions -- is not established here and remains an open question.

## 7. GR LIMIT FROM THE ACTION

### 7.1 The Two-Step Limit

Limit 1:  $\phi \rightarrow \phi_0$  (frozen,  $X_\phi = 0$ ) gives  $G_2 \rightarrow -V(\phi_0) = -\Lambda_{\text{obs}}*c^4/(8*\pi*G)$ ,  $G_3 \rightarrow 0$ ,  $G_4 \rightarrow M_{\text{Pl}}^2/2$ . Limit 2:  $\psi \rightarrow \infty$  (incoherent) gives  $K(\psi) \rightarrow 0$  and  $D(\psi) \rightarrow 0$ , decoupling psi and restoring  $L_{\text{matter}}(g_{\mu\nu})$ .

#### Theorem 7.1 -- GR Limit from the Action

$$S_{\text{SCT}} \rightarrow S_{\text{EH}} + S_{\text{Lambda}} + S_{\text{matter}} = S_{\text{GR}}[\text{Lambda}_{\text{obs}}] \quad (77)$$

Stronger than the algebraic demonstrations of P11 Section 3.4 and P12 Table 3: LCDM is a consistent sub-theory of SCT at the action level, not merely a special solution of the field equations.  $\text{Lambda}_{\text{obs}}$  emerges from  $V(\phi_0)$  as a classical derived quantity of order  $H_0^2$ . This provides a geometric origin for  $\text{Lambda}_{\text{eff}}$  but does not solve the cosmological constant problem: the smallness of  $\text{Lambda}_{\text{obs}}$  requires explaining why  $\alpha_n$  takes its observed value, which remains an open naturalness problem of equal difficulty. See Section 9.2.

## 8. PERTURBATION EQUATIONS FROM FIRST PRINCIPLES

### 8.1 Second-Order Action and Quadratic Lagrangian

Expanding  $S_{\text{SCT}}$  to second order in perturbations around the FRW background in Newtonian gauge with scalar field perturbations  $\delta\phi$ ,  $\delta\psi$ : The quadratic action produces coupling coefficients:  $\alpha_B = 2\dot{\phi}_0(\alpha_n/H_0\phi_0)/H$  (kinetic braiding),  $\alpha_D = \sqrt{2X_{\psi_0}}D'(\psi_0)\rho_b/M_{\text{Pl}}^2$  (disformal coupling). These are new terms absent from the P11 Section 8.2 by-analogy approach.

### 8.2 Modified Poisson and Growth Equations

In the sub-Hubble quasi-static limit, the modified Poisson equation is:

$$k^2\Phi/a^2 = -4\pi G [\rho_b\delta_b \mu_\phi(k,a) A(z) + \rho_r\delta_r + \text{correction from } \delta\psi] \quad (82)$$

with scale-dependent effective Newton's constant  $\mu_\phi(k,a) = 1 + \alpha_B^2/(Q_s^\phi + \alpha_B^2) * k^2/(k^2 + m_\phi^2 a^2)$ . The modified growth equation:

$$\delta\ddot{b} + (2H + \alpha_D\dot{A}/A)\delta\dot{b} - 4\pi G\rho_b A(z)\mu_\phi(k,a)\delta_b = S_{\text{cross}}(\delta\psi, k, a) \quad (84)$$

The leading-order limit recovers the P11 Section 8.2 by-analogy result as a theorem. The effective coupling and slip:

$$\mu_{\text{SCT}}(k,a) = A(z) * \mu_\phi(k,a) \quad (87a)$$

$$\eta_{\text{SCT}}(k,a) = \Phi/\Psi \text{ approximately } 1 + O(\alpha_B^2/M_{\text{Pl}}^2) \text{ approximately } 1 \quad (87b)$$

### Theorem 8.1 -- First-Principles Perturbation Equations

The first-principles perturbation equations confirm P11 Section 8.2 at leading order and add three corrections: scale-dependent braiding enhancement  $\mu_\phi(k,a)$  at approximately 1% at sub-Hubble scales; disformal Hubble friction  $\alpha_D \dot{A}/A$ ; scalar-matter cross-coupling  $S_{\text{cross}}$ . All corrections become relevant at horizon scales  $k \sim aH$  probed by Euclid and CMB-S4. The EFT parametrization  $\mu_{\text{SCT}} = A(z)\mu_\phi(k,a)$ ,  $\eta_{\text{SCT}}$  approximately 1 provides the theoretical structure for comparison to EFTCAMB and Euclid weak lensing data; quantitative comparison requires the full Boltzmann implementation of Paper 15.

## 9. OPEN QUESTIONS AND PATH TO PAPER 15

### 9.1 What This Paper Establishes

Four foundational results: (1) the unified SCT action  $S_{\text{SCT}}$  within the surviving Horndeski class; (2) the SCT-MASTER equation recovered as a variational theorem under three physically justified approximations ( $\xi \rightarrow 0$ , slow-roll, virialized  $\psi$ ); (3) the exponential decay law, coherence amplification  $A$ , and NIPOK rotation function  $G(r,t)$  derived simultaneously from the scalar field equations of motion; (4) the NIPOK metric proven unique on-shell given stationarity, axisymmetry, and SCT boundary conditions, retroactively upgrading P12 from construction to theorem within those conditions.

### 9.2 Honest Ledger of Remaining Open Tasks

$\alpha_n$  naturalness: Theorem 4.1 derives the exponential form of the decay law but the rate  $\alpha_n$  remains observationally constrained ( $\alpha_{\text{eff}} \sim 0.9-1.0 H_0$  from cluster velocity dispersion evolution). A first-principles calculation of  $\alpha_n$  from the N-body orbital decay rate requires the structure formation simulation of Paper 15. The smallness of  $\Lambda$  requires explaining why  $\alpha_n$  takes its observed value -- this is an open problem equivalent in difficulty to the original cosmological constant problem, honestly acknowledged here. The claim that SCT dissolves the fine-tuning problem is retracted; the problem is recast as an  $\alpha_n$  naturalness problem, not eliminated.

$\alpha_n$  parameter accounting. The rate  $\alpha_n$  is constrained by CMB power spectrum shape and expansion history data rather than derived from first principles within the current SCT framework. Specifically,  $\alpha_{\text{eff}} \sim 0.9-1.0 H_0$  is set by matching the observed cluster velocity dispersion evolution and the dark energy equation of state; it is not a prediction. This means that the  $\Lambda_{\text{eff}}$  evolution in SCT retains one observationally-fixed free parameter, analogous to the role of the cosmological constant in LCDM. The predictive cost of this parameter is explicitly accounted for in the Bayesian evidence calculation of Paper 16 (doi:10.13140/RG.2.2.10321.29288), which computes Bayesian odds using the observed  $\alpha_n$  value and penalizes SCT for the additional parameter relative to LCDM. The 44:1 Bayesian odds in SCT's favor are therefore computed after this penalty has been applied, not before. The cosmological constant problem is not solved by

the SCT framework but reframed: instead of asking why the vacuum energy takes its observed value, the question becomes why  $\alpha_n$  takes its observed value. Both are naturalness problems of comparable difficulty. Honest acknowledgment of this equivalence is required so that the reframing is not mistaken for a resolution.

Full SCT-MASTER derivation without approximations: Theorem 3.1 recovers the SCT-MASTER equation in the joint limit  $\xi \rightarrow 0$ , slow-roll, and virialized pocket. The behavior of the metric variation outside these limits -- including  $\xi$ -correction terms, the non-slow-roll sector, and non-virialized environments -- is not covered by this paper and is the primary subject of Paper 19.

Rotating sector perturbations: Section 8 covers the  $J = 0$  FRW limit. Vector perturbations from the off-diagonal NIPOK components  $F(r,t)$  -- relevant for the hemispherical power asymmetry prediction -- require expanding  $S_{\text{SCT}}$  to second order around the rotating NIPOK background. Deferred to Paper 15.

Chi-squared comparison to data: the modified CAMB/CLASS implementation recipe is fully specified (P11 Section 8.6 plus three additions from Section 8.8). Running this and computing chi-squared against Planck 2018  $C_l$  spectra and DESI 2024 BAO data is the primary quantitative deliverable of Paper 15. Preliminary Bayesian model comparison between SCT and LCDM, using the observed parameter values without the full Boltzmann run, is reported in Paper 16 (doi:10.13140/RG.2.2.10321.29288), which finds odds of 44:1 in favor of SCT. That result is cited here rather than reproduced; the definitive chi-squared comparison awaits Paper 15.

Quantization: the SCT action is classical. Loop corrections and renormalization group running of the Horndeski functions are beyond the current scope. Known issues with quantum stability of the Horndeski class (radiative corrections to  $G_4$  may reintroduce  $G_{4X}$ ) would require a UV completion not pursued here.

### 9.3 Three New Falsifiable Forecasts

Forecast 1 -- Scale-dependent effective Newton's constant:  $\mu_{\text{SCT}}(k,a) = A(z)^*[1 + \alpha_B^2/(Q_s^\phi + \alpha_B^2) * k^2/(k^2 + m_\phi^2 a^2)]$  from Equation (87a). This is a theoretical forecast; quantitative comparison to Euclid weak lensing cross-correlated with DESI galaxy clustering requires full Boltzmann implementation (Paper 15). The braiding parameter  $\alpha_B \sim \mathcal{O}(2)$  is independently constrained from dark energy evolution.

Forecast 2 -- Rotation-dark matter correlation: pockets with larger  $A$  (stronger dark matter analog, higher mass-to-light ratios) have larger  $J_{\text{pocket}}$  since both are sourced by the same  $\psi$ -field. This forecasts a correlation between the RAR normalization and bulk rotational velocity of galaxy groups, testable with joint DESI kinematics and weak lensing pending full implementation.

Forecast 3 -- Gravitational slip  $\eta_{\text{SCT}}$  approximately 1: the gravitational slip is unity at leading order, distinguishing SCT from theories where  $\eta$  departs significantly from unity.

This is a theoretical forecast; quantitative comparison to CMB-S4 lensing combined with galaxy surveys requires full Boltzmann implementation (Paper 15).

## 10. CONCLUSION

This paper has supplied the missing variational foundation of Successive Collision Theory. The unified action  $S_{\text{SCT}}$ , identified within the two-scalar surviving post-GW170817 Horndeski class, produces four theorems that close the two foundational criticisms raised by peer review and complete the analytical program of the SCT series.

The SCT-MASTER equation is recovered from variation of  $S_{\text{SCT}}$  under three physically justified approximations:  $\xi \rightarrow 0$ , quasi-static slow-roll for  $\phi$ , and virialized pocket for  $\psi$  (Theorem 3.1). These approximations are well-satisfied at the scales where SCT is applied. The full derivation without these approximations is the subject of Paper 19. The exponential decay law for  $\Lambda_{\text{parent}}$ , the coherence amplification factor  $A$ , and the NIPOK rotation function  $G(r,t)$  are proven simultaneously as consequences of the scalar field equations of motion (Theorems 4.1, 4.2, 4.3) -- three physical outputs of one variational principle. The NIPOK metric is proven to be the unique stationary axisymmetric solution of the SCT field equations in the presence of nonzero  $\psi$ -field angular momentum, given the conditions of stationarity, axisymmetry, and the SCT pocket boundary conditions (Theorem 6.1). Uniqueness under relaxed assumptions remains open. This retroactively upgrades Paper 12 from construction to derived theorem within the stated conditions.

The SCT action passes all eight physical viability conditions. The GR limit is established at the action level (Theorem 7.1), stronger than prior demonstrations. First-principles perturbation theory confirms the leading-order results of P11 Section 8.2 and provides three systematic corrections in the EFT of dark energy parametrization. Quantitative comparison to Euclid and CMB-S4 data requires the full Boltzmann implementation of Paper 15. Preliminary Bayesian model comparison to  $\Lambda$ CDM is reported in Paper 16 (doi:10.13140/RG.2.2.10321.29288), which finds Bayesian odds of 44:1 in SCT's favor using observed parameter values. The question of whether SCT provides a competitive fit to the full Planck 2018 and DESI 2024 dataset in a complete chi-squared analysis -- the decisive empirical test -- is the subject of Paper 15.

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